Worksheet for November 21

Problems marked with an asterisk are to be placed in your math diary.

(1.*) Suppose $\mathbf{F} = x\vec{i} + y\vec{j} + (z-2)\vec{k}$. Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, for S the helicoid with parameterization $G(u,v) = (u\cos(v), u\sin(v), v)$, with $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.

(2.*) Let $\mathbf{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S denote the sphere of radius R centered at the origin. Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ with respect to the unit outward normal in two ways: First by parameterizing S and second, without parameterizing S.

(3.*) Suppose S is the graph of $z = f(x,y)$ defined over the domain $D \subseteq \mathbb{R}^2$. Let $h(x,y,z)$ be a scalar function defined on S and $\mathbf{F}(x,y,z) = F_1(x,y,z)\vec{i} + F_2(x,y,z)\vec{j} + F_3(x,y,z)\vec{k}$ be a vector field defined on S. Show that:

(i) $\int\int_S h(x,y,z)\; dS = \int\int_D h(x,y,f(x,y))\sqrt{1 + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}\; dxdy$.

(ii) $\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int_D -F_1(x,y,f(x,y))\frac{\partial f}{\partial x} - F_2(x,y,f(x,y))\frac{\partial f}{\partial y} + F_3(x,y,f(x,y))\; dxdy$,

where (ii) is taken with respect to the upward normal vector.